Phase-space simulation methods for quantum dynamics

S. Hoffmann, M. R. Dowling, P. Deuar, A. S. Bradley,
J. F. Corney, M. K. Olsen, M. J. Davis, and P. D. Drummond
ACQAO, School of Physical Sciences, The University of Queensland, QLD 4072, Australia

The development of tractable methods to simulate the dynamics of strongly correlated quantum systems of bosons and fermions is the goal of this research stream in quantum phase-space methods. The methods are based on the non-classical, positive-P techniques that were originally developed in quantum optics and are now being tailored to systems of massive particles, i.e., electrons, atoms or molecules. The previous methods can be extended and improved through stochastic gauges or novel basis sets.

The gauge techniques are able to deal with extremely complex irreversible master equations as well as unitary dynamics. The application of stochastic gauge representations to the general case of interacting bosonic systems has been analysed in depth in recent papers[1]. Using stochastic gauges that give real weights, we have been able to apply Monte Carlo techniques such as the Metropolis algorithm to real-time quantum dynamics[2].

Simulation methods based on a generalised Gaussian representation are a powerful alternative to coherent-state methods. These novel techniques are being tested on dynamical simulations of atoms and molecules in optical lattices. Examples include simple models of the molecular association and dissociation of fermionic and bosonic atoms, as well as tunnelling dynamics in double-well potentials[3].

The truncated Wigner method has been found to give predictions very close to those of an exact matrix method for tunnelling in a double well potential where reasonable numbers of atoms are involved. However, it was found that for photoassociation in an optical lattice, the truncated Wigner can be inaccurate for low occupation numbers. This work is being extended to find more accurate methods[4].

We are also developing a hybrid simulation method that combines the best features of the positive-P and truncated Wigner methods. The intended applications are to atomic Bose-Einstein condensate systems, atom lasers produced from those BECs and various outcoupling and detection scenarios. In systems with a small number of highly occupied modes—the condensed modes—and a large number of modes with small occupation numbers, the Wigner representation should work well for the condensed modes while the positive-P representation should be successful with the other modes. The formalism has been developed and has been applied to simple reservoir models to test the behaviour of the sampling error in a multimode system[5].

In collaboration with L. I. Plimak and W. Schleich (Ulm) and M. Fleischhauer (Kaiserslautern), we have developed a method to calculate time-normally ordered correlation functions in the Wigner representation. This uses Kubo's linear response relation to calculate the expectation values of two-time commutators[6].

References

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